1. For each of the following single variable functions, find (i) \( \frac{dy}{dx} \) and (ii) the extreme values and indicate whether it is a maximum or minimum:

   (a) \( y = 9 - x^2 \)
   (b) \( y = 3x^3 - 2x - 39 \)
   (c) \( y = 8x^{1/2} - 4x + 2 \)

2. Consider the following implicit function, where \( y \) is the endogenous variable and \( x_1, x_2 \) are the exogenous variables.

\[
F(y; x_1, x_2) = \frac{1}{2} y - 2x_1 + 4x_1x_2 = 0
\]

Compute \( M_y/M_{x_1} \) and \( M_y/M_{x_2} \) using (i) the direct (explicit) approach and (ii) the implicit function approach.

3. Consider the following matrices:

\[
A = \begin{pmatrix} 6 \\ 3 \end{pmatrix}, B = \begin{pmatrix} -2 & 3 \\ -4 & 5 \end{pmatrix}, C = \begin{pmatrix} 1 & 0 \\ 0 & 8 \end{pmatrix}, \text{ and } D = \begin{pmatrix} 0 & 1 & 4 \\ 3 & 2 & 0 \\ -1 & 1 & 0 \end{pmatrix}
\]

Compute the following (or indicate if they do not exist):

   (a) The determinants of \( A, B, C, \text{ and } D \)
   (b) \( A+B \) and \( B+C \)
   (c) \( BxA, BxC, \text{ and } BxD \)

4. Solve the following system of two equations and two unknowns by (i) direct method and (ii) Cramer’s Rule:

\[
\begin{align*}
  x_1 + 2x_2 &= 6 \\
  3x_1 - 2x_2 &= 10
\end{align*}
\]
5. Consider the following model of demand and supply. The endogenous variables are quantity \( Q \) and price \( P \) and let the exogenous variable be consumer income \( I \). The equilibrium \( (Q,P) \) must satisfy the following functions:

\[
Q = D(P, I) = 3 + \frac{I}{P} \quad \text{(Demand Curve)}
\]

\[
Q = S(P, I) = 4P \quad \text{(Supply Curve)}
\]

Compute the impact of an exogenous change in \( I \) on the equilibrium quantity and price (i.e., \( \frac{dQ}{dI} \) and \( \frac{dP}{dI} \)) using Cramer’s Rule.

6. Find the extreme values of the following multi-variable functions:

(a) \( \max f(x_1, x_2) = 12x_1 - 4x_2 - 2x_1^2 + 2x_1x_2 - x_2^2 \)

(b) \( \min f(x_1, x_2, x_3) = 29 - (x_1^2 + x_2^2 + x_3^2) \)

7. Solve the following constrained optimization problems using (i) the chain-rule approach and (ii) the Lagrangian approach:

(a) Maximize \( U(x_1, x_2) = x_1x_2 + 2x_1 \) subject to \( 4x_1 + 2x_2 = 60 \)

(b) Maximize \( f(x_1, x_2) = x_1x_2 \) subject to \( x_1 + x_2 = 6 \)